**Linear Programming: Introduction**

Optimization problems are present in our daily lives, stemming from our need to make the right decisions. For example, which route to take to Deakin, or how many hours to allocate to studies.

The decisions in an optimization problem are often represented as the symbols .

In many optimization problems, we are faced with restrictions or constraints.

The generally ways to express constraint optimization are:

* Less or equal to a constraint:
* More or equal to a constraint:
* Equal to a constraint:

There is a goal or objective that the decision maker considers when deciding which action is best.

The mathematical formulation can be written as:

Max or min

Subject to

…

Linear programming (LP) involves creating and solving optimization problems with linear objective functions and linear constraints only.

We can rewrite the formula as:

Max or min

Subject to

…

**Linear Programming: A Graphical Approach**

We look at a certain case:

Blue Ridge Hot Tubs manufactures and sells two models of hot tubs: the Aqua-Spa and the Hydro-Lux. Howie Jones, the owner and manager of the company, needs to decide how many of each type of hot tub to produce during his next production cycle. Howie buys prefabricated fiberglass hot tub shells from a local supplier and adds the pump and tubing to the shells to create his hot tubs. (This supplier has the capacity to deliver as many hot tub shells as Howie needs.) Howie installs the same type of pump into both hot tubs. He will have only 200 pumps available during his next production cycle. From a manufacturing standpoint, the main difference between the two models of hot tubs is the amount of tubing and labor required. Each Aqua-Spa requires 9 hours of labor and 12 feet of tubing. Each Hydro-Lux requires 6 hours of labor and 16 feet of tubing. Howie expects to have 1,566 production labor hours and 2,880 feet of tubing available during the next production cycle. Howie earns a profit of $350 on each Aqua-Spa he sells and $300 on each Hydro-Lux he sells. He is confident that he can sell all the hot tubs he produces. The question is, how many Aqua-Spas and Hydro-Luxes should Howie produce if he wants to maximize his profits during the next production cycle?

Max

Subject to

We are interested in finding the values and .

By graphing all the formulas, we get:

A graph of a line

Description automatically generated

As we keep adding formulas, the feasible region (where optimal values of and lie) will decrease.

Issues in linear programming include: alternate optimal solutions, redundant constraints, unbounded solution and infeasibility. Unbounded solutions and infeasibility prevent us from solving an LP model whereas the rest are just anomalies.

A graph of functions and functions

Description automatically generated

As you can see in this graph, as the level curves shifts farther away from the origin, the objective function increases. The feasible region is not bounded to the origin so you can make the objective function infinitely large. This is called an unbounded solution.

An LP problem is infeasible if there is no way to satisfy all the constraints simultaneously. For example:

A graph of a function

Description automatically generated

**Linear Programming: Standard and Slack Forms**

Various algorithms for linear problems needs objective function and constraints in a certain form.

In the standard form, all the constraints are in inequalities.

In the slack form all constraints must be in equalities.

An optimization problem in standard form looks like:

Maximize

Subject to for i=1,2,…,m

for j=1,2,…,n

The formula for the slack form is:

, where

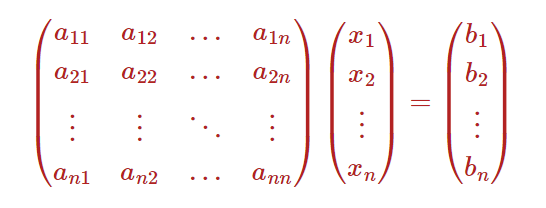
**Linear Programming: Gaussian Elimination**

We define a system of linear equations as:

…

We can write this in matrix-vector notation:

Ax = b



If A is non-singular, it has an inverse , which can be computed by .

LU-Decomposition is recommended to calculate inverse

In LU decomposition, we find two nxn matrices L and U such that A=LU.

L is a lower-triangular matrix

U is an upper-triangular matrix

We can rewrite Ax=b as LUx=b

Forward substitution solves the lower triangular system first for unknown y is calculated by Ly=b, where y=Ux

Backward substitution solves the upper triangular system for unknown x is calculated by Ux=y.

**Linear Programming: Simplex Algorithm**

To execute the simplex method, we need to:

* Check if the linear problem is a standard maximization problem in standard form
* Create slack variables
* Create a system of equations using the variables
* Place the equations into a matrix, with the objective equation in the bottom row
* Select a pivot column by finding the most negative indicator
* Select a pivot row
* Find pivot
* If we don’t get all non-negative indicators, repeat step 5 and 7

A paper with text and images

Description automatically generated

A hand holding a piece of paper with writing on it

Description automatically generated